

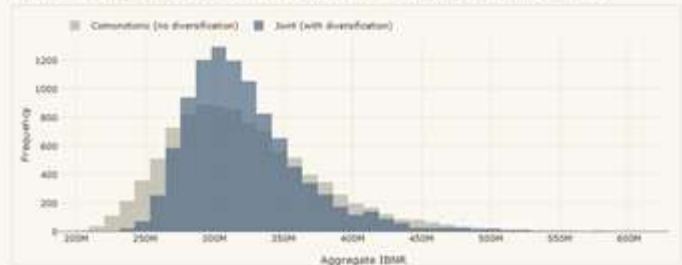
Measuring Correlations and Aggregating Model Results

Aggregate IBNR by Accident Year

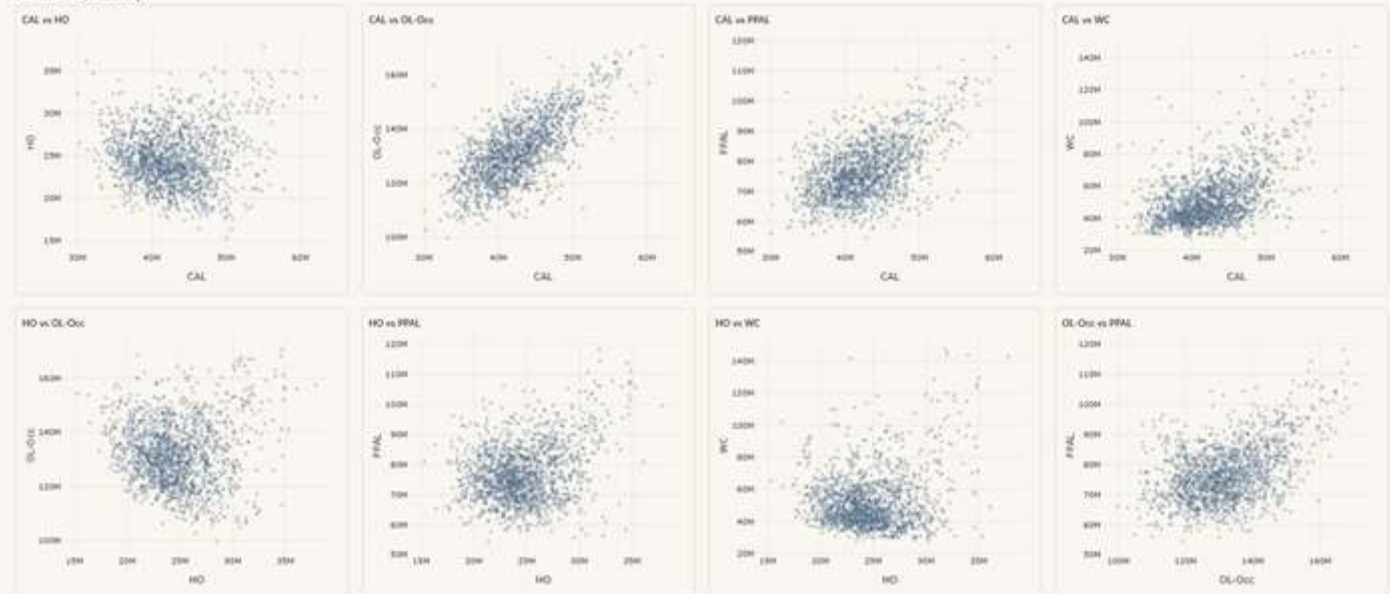
AY	Mean	SE	50th %ile	75th %ile	90th %ile	95th %ile	99th %ile	99.5th %ile
2016	3,214,447	873,181	3,006,545	3,572,426	4,363,152	4,976,448	6,606,660	7,314,179
2017	4,216,476	1,138,513	3,386,603	4,643,398	5,335,520	6,264,639	8,183,833	9,254,642
2018	6,306,000	1,656,967	6,044,641	7,091,788	8,360,325	9,369,060	12,231,506	13,665,100
2019	8,454,930	2,001,431	8,036,796	9,194,340	10,785,416	12,054,717	15,447,324	17,113,464
2020	10,617,954	2,437,949	10,475,057	11,899,832	13,837,048	15,380,320	19,490,247	21,409,190
2021	16,656,180	3,827,687	16,014,917	18,033,234	20,660,769	22,701,309	28,307,390	30,948,719
2022	26,536,901	4,733,681	25,670,992	28,592,371	32,274,410	35,044,317	43,181,627	45,981,263
2023	43,409,476	6,800,415	42,248,185	46,572,771	51,761,340	55,861,910	66,528,217	70,947,168
2024	68,312,447	9,452,187	66,881,361	72,591,707	79,868,484	85,607,406	100,091,331	105,974,190
2025	134,325,984	14,726,952	131,862,963	140,969,011	152,416,834	161,013,728	184,415,558	194,318,611
Total	322,468,821	43,723,398	314,231,588	342,968,207	377,417,213	408,126,527	481,257,695	511,318,097

Aggregate Distribution — Diversification Benefit

Joint (square-rooted) aggregate vs. the fully correlated upper bound. The joint distribution's lighter right tail is the diversification benefit.



Pairwise Dependency



Jim Bedford, FCAS, MAAA
Bedford Actuarial Advisory, LLC

BASR produces the full predictive distribution of unpaid claims, by accident period and in total, with percentiles, reserve ranges, a one-year claims development result (CDR), and insights into historical trends, valuable not only to the reserving process but also to ratemaking, risk management, and capital modeling.

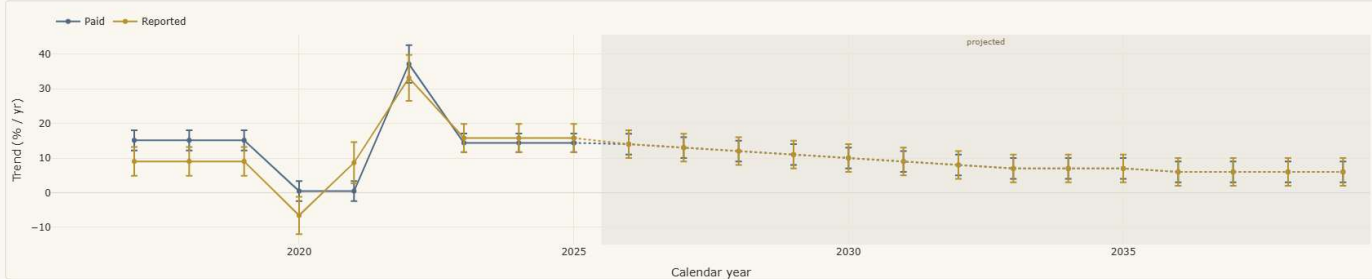
For individual segments, the user can select the paid model, the reported model, or a combination of the two. This is illustrated on the next page.

BASR measures correlations between segments and allows the user to select the copula form they find most appropriate, illustrated on the next two pages.

Aggregate distributions can then be simulated, reflecting diversification benefits and showing modeled dependencies between segments, illustrated on the last two pages.

Calendar-Year Trend: Paid vs Reported

Each model's calendar-year trend (% per year, ± 5E): observed (solid) on the unshaded years, the projected trend it carries forward (dashed) on the shaded future years.

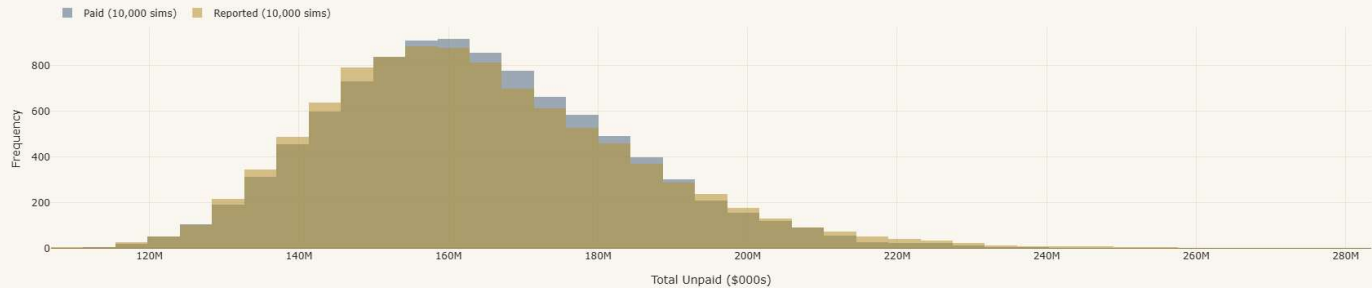


Distribution of Unpaid Losses by Accident Year (\$000s)

Both sides shown on a comparable unpaid basis. Paid's reserve is already unpaid (ultimate – paid); Reported is IBNR + case reserves (= ultimate – paid), so each Reported percentile is its IBNR percentile shifted by the deterministic case amount. For the lognormal family the 50th–95th %ile are the analytic (method-of-moments) values and the 99th / 99.5th need a simulation; other families have no analytic percentiles, so every percentile comes from each side's Monte Carlo. Once a side's simulation is run, all of its percentiles read from it, matching that tab.

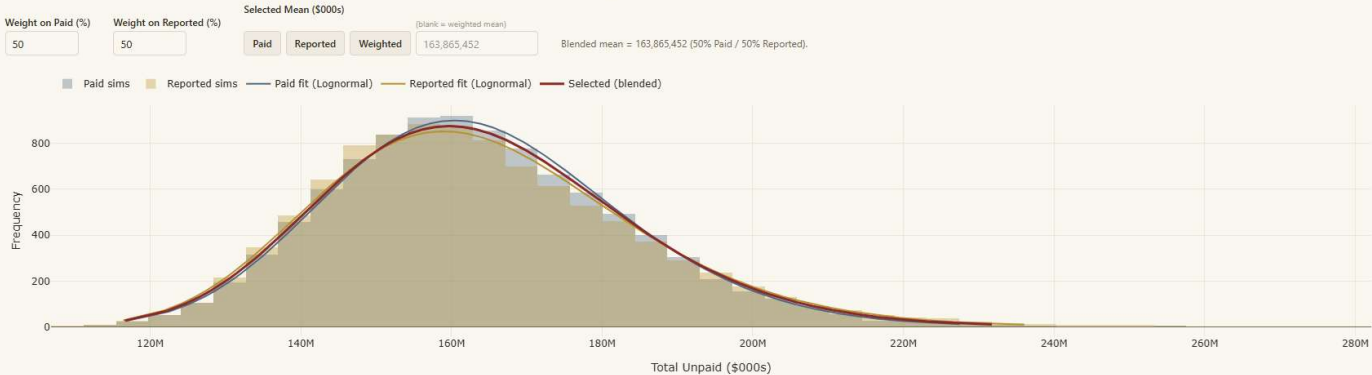
AY	Paid — Unpaid								Reported — Unpaid (IBNR + Case)							
	Mean	50th %ile	75th %ile	90th %ile	95th %ile	99th %ile	99.5th %ile		Mean	50th %ile	75th %ile	90th %ile	95th %ile	99th %ile	99.5th %ile	
2016	1,402,883	1,391,564	1,519,424	1,642,620	1,722,871	1,885,383	1,952,894		1,904,203	1,893,777	2,003,648	2,112,160	2,182,159	2,329,481	2,380,427	
2017	2,431,904	2,418,163	2,618,946	2,817,764	2,948,756	3,192,654	3,302,453		3,010,428	2,991,088	3,157,455	3,321,735	3,428,606	3,653,998	3,740,232	
2018	4,105,815	4,074,395	4,404,782	4,721,370	4,920,600	5,337,671	5,493,453		4,773,237	4,737,728	5,037,261	5,342,736	5,534,710	5,961,040	6,160,623	
2019	6,097,266	6,050,504	6,515,287	6,979,434	7,294,012	7,905,144	8,115,899		6,725,277	6,666,441	7,123,620	7,592,248	7,900,541	8,570,979	8,899,244	
2020	8,497,927	8,431,294	9,076,662	9,716,577	10,147,217	10,998,140	11,340,334		8,996,555	8,897,300	9,574,804	10,244,348	10,721,994	11,723,195	12,074,139	
2021	13,095,082	12,990,135	13,979,597	14,930,348	15,581,568	16,883,090	17,531,831		13,677,501	13,513,132	14,606,777	15,681,726	16,475,553	18,168,897	18,760,338	
2022	19,516,464	19,341,212	20,866,356	22,375,775	23,440,305	25,570,485	26,278,363		20,891,023	20,615,585	22,342,598	24,183,815	25,334,934	27,977,243	29,130,859	
2023	27,797,580	27,531,921	29,895,675	32,092,924	33,638,704	36,809,043	38,089,748		29,940,278	29,485,580	32,154,534	35,002,689	36,849,151	41,189,165	42,717,459	
2024	36,158,622	35,722,799	39,108,317	42,378,531	44,407,137	49,270,403	51,411,032		35,051,491	34,471,803	37,979,754	41,664,859	44,121,200	49,966,147	51,734,567	
2025	44,714,505	44,073,076	48,755,079	53,346,223	56,237,491	62,770,656	65,367,660		38,885,561	38,078,533	42,853,091	47,710,846	51,069,935	59,103,425	62,180,071	
Total	163,818,047	161,970,121	176,066,483	189,398,644	198,373,872	217,605,020	225,470,829		163,855,555	161,472,613	176,341,816	191,853,843	202,047,457	226,907,354	235,582,645	

Total Unpaid Distribution — Paid vs Reported



Selected Distribution — Weighted Paid / Reported

Each simulation's best-fitting distribution (by AIC), blended percentile-by-percentile with the weight below; Reported is shifted by total case onto the unpaid basis before blending. The blended mean equals the weighted average of the two means unless a Selected Mean is entered, which re-centers the blended distribution without changing its shape. These selections carry into the Copula tab's Selected Distributions aggregation.



Candidate fits by AIC — Paid: Lognormal 363,823 (best) · Gamma 363,916 · Normal 364,249 · Reported: Lognormal 365,034 (best) · Gamma 365,201 · Normal 365,818

Unpaid Losses	Distribution	Parameters	Mean	SD	CV	50th %ile	75th %ile	90th %ile	95th %ile	99th %ile	99.5th %ile
Paid	Simulated	—	163,818,047	19,645,843	12.0%	161,970,121	176,066,483	189,398,644	198,373,872	217,605,020	225,470,829
	Fitted	Lognormal $\mu=18,907, \sigma=0.118$	163,789,795	19,433,290	11.9%	162,648,968	176,150,983	189,258,996	197,565,632	214,143,465	220,554,142
Reported	Simulated	—	163,855,555	21,248,624	13.0%	161,472,613	176,341,816	191,853,843	202,047,457	226,907,354	235,582,645
	Fitted	Lognormal $\mu=18,907, \sigma=0.127$	163,941,110	20,842,473	12.7%	162,264,835	176,831,381	191,383,770	200,807,058	220,061,819	227,662,869
Selected	Blended	$\mu=18,907, \sigma=0.121$	163,865,452	19,923,085	12.2%	162,456,901	176,491,182	190,321,383	199,186,345	217,102,642	224,108,506

Fitted vs Simulated columns show how closely each parametric fit tracks its simulation — the quality check behind the blend.

Copula shape explorer

Set the correlation and copula parameters to see the resulting dependence shape. This is illustrative only; it does not read the data or change any model selection. The actual aggregation models and their parameters are set below.

Dependence Copula ⓘ

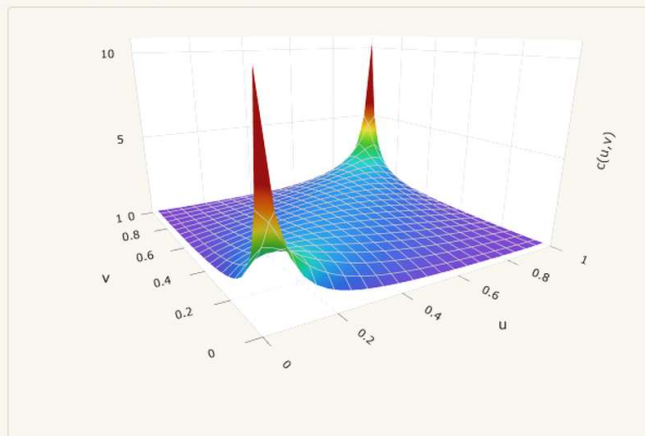
Student's t-copula (tail-dependent) ▼

Degrees of freedom (ν) ⓘ

5

Correlation (ρ): 0.60

$c(u,v)$ = density of the assumed copula on percentile axes (u, v); a value of 6.0 means that specific combination is 6.0 times more likely to happen than if the two variables were independent.

Student's t-copula (tail-dependent) — $df(\nu) = 5$ — Correlation $\rho = 0.60$ 

Copula shape explorer

Set the correlation and copula parameters to see the resulting dependence shape. This is illustrative only; it does not read the data or change any model selection. The actual aggregation models and their parameters are set below.

Dependence Copula ⓘ

Skew-t copula (asymmetric tail) ▼

Degrees of freedom (ν) ⓘ

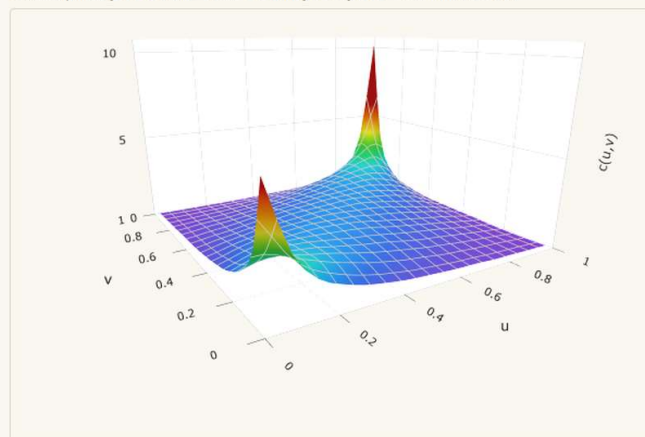
5

Tail asymmetry ⓘ

0.3

Correlation (ρ): 0.60

$c(u,v)$ = density of the assumed copula on percentile axes (u, v); a value of 6.0 means that specific combination is 6.0 times more likely to happen than if the two variables were independent.

Skew-t copula (asymmetric tail) — $df(\nu) = 5$, Tail asymmetry 0.30 — Correlation $\rho = 0.60$ 

Copula shape explorer

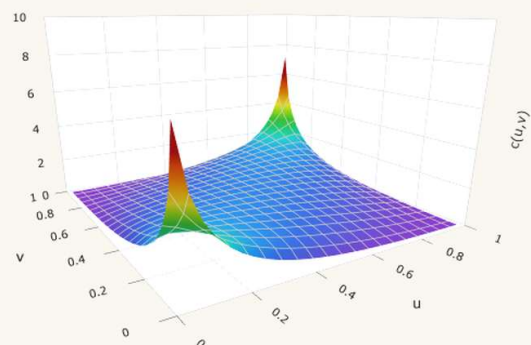
Set the correlation and copula parameters to see the resulting dependence shape. This is illustrative only; it does not read the data or change any model selection. The actual aggregation models and their parameters are set below.

Dependence Copula ⓘ

Gaussian copula ▼

Correlation (ρ): 0.60

$c(u,v)$ = density of the assumed copula on percentile axes (u, v); a value of 6.0 means that specific combination is 6.0 times more likely to happen than if the two variables were independent.

Gaussian copula — Correlation $\rho = 0.60$ 

Copula shape explorer

Set the correlation and copula parameters to see the resulting dependence shape. This is illustrative only; it does not read the data or change any model selection. The actual aggregation models and their parameters are set below.

Dependence Copula ⓘ

Skew-t copula (asymmetric tail) ▼

Degrees of freedom (ν) ⓘ

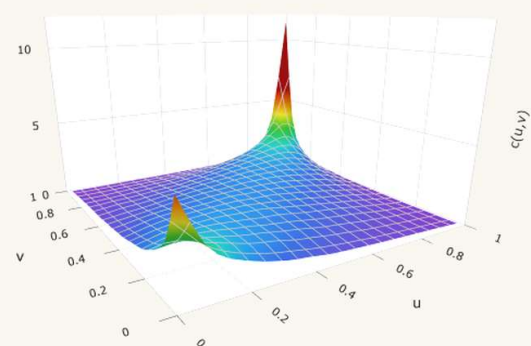
5

Tail asymmetry ⓘ

0.5

Correlation (ρ): 0.60

$c(u,v)$ = density of the assumed copula on percentile axes (u, v); a value of 6.0 means that specific combination is 6.0 times more likely to happen than if the two variables were independent.

Skew-t copula (asymmetric tail) — $df(\nu) = 5$, Tail asymmetry 0.50 — Correlation $\rho = 0.60$ 

Data

Barnett-Zehnwirth (Lognormal PTF)

GLM Models

Paid Loss

Reported Loss

Comparison

Copula

P&C Industry — Copula Aggregation (Multi-LOB) — Reported ⓘ

Lines of Business (pick 2 or more)

CAL

HO

OL-Occ

PPAL

WC

Dependence Copula ⓘ

Skew-t copula (asymmetric tail)

df (v) ⓘ

5

Tail asymmetry ⓘ

0.5

Simulations ⓘ

10,000

Seed (0 = Random) ⓘ

42

Run Aggregation

Correlation Matrix (estimated from residuals on 41 common cells)

Closer to 0 = more diversification benefit; closer to 1 = lines move together (less benefit).

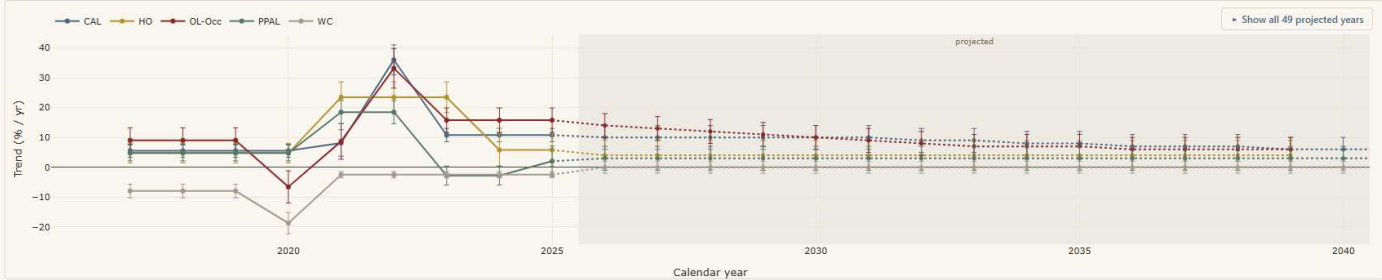
	CAL	HO	OL-Occ	PPAL	WC
CAL	1.000	-0.057	0.695	0.427	0.408
HO	-0.057	1.000	-0.112	0.195	-0.036
OL-Occ	0.695	-0.112	1.000	0.439	0.343
PPAL	0.427	0.195	0.439	1.000	0.383
WC	0.408	-0.036	0.343	0.383	1.000

Per-Line Standalone Distributions

Line	Family	n obs	k params	LLR	BIC	AICc	Mean IBNR	SD	95th %ile	99.5th %ile
CAL	lognormal	55	15	69.45	-78.80	-96.60	42,413,521	4,563,546	50,389,126	56,502,922
HO	lognormal	41	10	19.97	-2.80	-12.60	24,430,504	3,206,526	30,083,670	33,982,437
OL-Occ	lognormal	55	12	73.64	-99.18	-115.84	127,655,524	24,438,187	172,291,377	210,531,547
PPAL	lognormal	55	13	59.41	-66.73	-83.95	76,316,201	9,232,252	92,867,888	107,662,441
WC	lognormal	55	13	52.56	-53.02	-70.24	51,653,071	16,529,845	81,610,507	123,660,762

Calendar-Year Trend by Line

Each line's calendar-year trend (% per year, ± SE): observed (solid) on the unshaded years, the projected trend it carries forward (dashed) on the shaded future years.



Aggregate IBNR by Accident Year

AY	Mean	SE	50th %ile	75th %ile	90th %ile	95th %ile	99th %ile	99.5th %ile
2016	3,214,447	973,381	3,006,565	3,572,426	4,363,152	4,978,448	6,609,660	7,514,178
2017	4,219,476	1,139,513	3,986,683	4,645,998	5,539,620	6,264,409	8,163,633	9,204,642
2018	6,399,003	1,656,987	6,044,841	7,031,786	8,360,325	9,369,065	12,231,506	13,665,195
2019	8,424,950	2,001,431	8,036,706	9,194,540	10,785,418	12,054,717	15,447,324	17,115,464
2020	10,937,954	2,437,949	10,475,057	11,899,832	13,837,046	15,380,920	19,490,247	21,409,195
2021	16,658,132	3,327,687	16,014,917	18,033,234	20,660,769	22,701,309	28,207,390	30,849,713
2022	26,536,951	4,733,681	25,670,992	28,592,371	32,274,410	35,044,317	43,193,027	45,981,263
2023	43,439,478	6,800,415	42,248,185	46,572,771	51,761,340	55,861,910	66,528,217	70,947,168
2024	68,312,447	9,452,197	66,691,361	72,593,707	79,869,484	85,687,406	100,091,331	105,974,195
2025	134,325,984	14,726,950	131,862,985	140,988,011	152,416,934	161,013,728	184,419,558	194,518,811
Total	322,468,821	45,723,398	314,251,589	342,069,207	377,417,213	408,128,527	481,257,695	511,318,097

Means add to the total; percentiles do not (the Total row shows the aggregate distribution's own percentiles).

Expected Reporting Stream (by future calendar year)

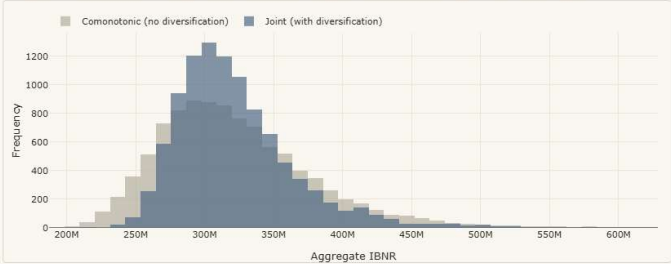
Aggregate IBNR emerging in each future calendar year — the projected payout pattern (for discounting / liquidity).

CY	Mean	SE	50th %ile	75th %ile	90th %ile	95th %ile	99th %ile	99.5th %ile
2026	121,303,921	5,998,075	120,536,175	124,556,086	129,083,146	132,133,510	139,352,713	142,160,900
2027	62,136,773	4,899,892	61,634,600	65,004,922	68,515,643	70,883,841	75,778,780	78,096,198
2028	38,240,161	4,637,428	37,697,892	40,908,555	44,367,419	46,595,624	51,508,115	53,499,255
2029	24,314,500	4,055,256	23,792,695	26,583,356	29,617,551	31,708,504	36,280,174	37,896,675
2030	16,268,213	3,511,565	15,738,310	18,172,770	20,864,718	22,814,760	26,776,832	28,849,223
2031	11,292,952	2,989,106	10,795,206	12,836,653	15,153,600	16,832,191	20,519,900	22,309,257
2032	8,230,359	2,559,362	7,767,587	9,481,164	11,533,155	12,945,147	16,552,919	17,898,811
2033	6,190,048	2,200,683	5,756,082	7,204,046	8,960,341	10,283,139	13,507,018	15,084,602
2034	4,851,943	1,925,826	4,438,247	5,673,298	7,214,461	8,430,388	11,343,809	12,953,211
2035	3,916,241	1,705,973	3,530,052	4,584,938	5,945,243	7,002,809	9,921,795	11,495,744
2036	3,107,542	1,449,926	2,772,782	3,625,323	4,818,512	5,748,791	8,308,549	9,714,664
2037	2,442,185	1,203,414	2,154,501	2,848,086	3,813,619	4,617,030	6,810,513	8,272,153
2038	1,897,022	969,230	1,670,877	2,222,577	2,963,406	3,623,498	5,405,297	6,596,458
2039	1,465,007	757,122	1,296,974	1,712,831	2,299,034	2,829,082	4,257,076	5,136,206
2040	1,132,651	597,586	1,000,399	1,349,177	1,819,633	2,175,244	3,307,473	3,795,997
+ Show all 49 periods								
Total	322,468,821	45,723,398	314,251,589	342,069,207	377,417,213	408,128,527	481,257,695	511,318,097

Means add to the total; SE and percentiles do not (the Total row shows the aggregate distribution's own figures).

Aggregate Distribution — Diversification Benefit

Joint (copula-coupled) aggregate vs. the fully correlated upper bound. The joint distribution's tighter right tail is the diversification benefit.



Tail-Risk Contribution (TVaR allocation)

Each line's expected unpaid in the worst (1 – q) of aggregate outcomes (co-TVaR / Euler allocation).

Line	TVaR 75%	TVaR 90%	TVaR 95%	TVaR 97.5%	TVaR 99%	TVaR 99.5%
CAL	85,499,449	88,085,812	89,825,535	91,555,044	94,352,167	96,254,641
HO	44,822,764	46,021,540	47,446,987	48,955,576	51,093,491	53,005,645
OL-Occ	194,393,853	209,965,317	221,682,827	233,827,245	250,356,801	262,917,816
PPAL	177,353,500	182,887,622	187,458,758	192,319,202	198,752,071	202,735,937
WC	106,933,137	120,805,757	132,953,144	147,032,929	167,712,885	185,095,573
Aggregate	609,002,703	647,766,048	679,367,251	713,689,996	762,267,415	800,009,611

Diversified Capital

Capital = the 99% / 99.5% measure in excess of the mean. The joint book needs less than the sum of standalone lines — that gap is the diversification credit.

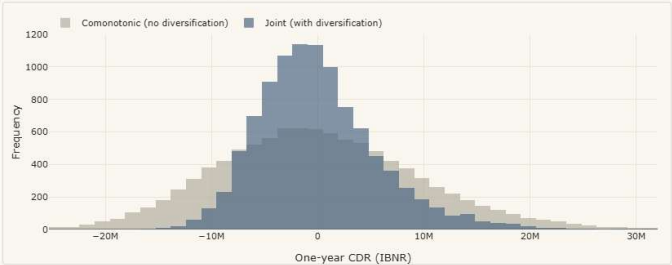
Capital basis	Sum of standalone	Joint (diversified)	Diversification credit
VaR 99 – mean	179,113,751	158,788,874	11.3%
TVaR 99 – mean	232,957,815	213,543,378	8.3%
VaR 99.5 – mean	209,871,289	188,849,276	10.0%
TVaR 99.5 – mean	272,408,971	251,285,574	7.8%

Data	Barnett-Zehnwirth (Lognormal PTF)	GLM Models
Paid Loss	Reported Loss	Comparison
	Copula	

One-Year Reserve Risk (CDR)

How much the combined reserve estimate can move after one more calendar year of experience — the Solvency II / ORSA one-year reserve-risk input, coupled across lines. The no-refit Merz-Wüthrich CDR equals next year's simulated diagonal (CY 2026), recentred to zero.

AY	Expected Rept. ⓘ	CDR SD ⓘ	0.5% ⓘ	50% ⓘ	99.5% ⓘ
2016	475,959	55,702	-110,662	—	193,285
2017	713,474	70,133	-147,231	—	230,375
2018	1,152,067	103,168	-233,298	—	303,820
2019	1,734,528	140,596	-330,588	—	415,965
2020	2,474,996	197,255	-450,635	—	549,797
2021	4,385,123	280,548	-664,971	—	793,466
2022	7,856,342	475,858	-1,094,847	—	1,380,156
2023	14,453,145	804,550	-1,894,189	—	2,265,734
2024	24,730,782	1,332,021	-2,999,334	—	4,038,566
2025	63,488,058	4,228,926	-9,144,533	—	13,716,978
Total	121,464,475	5,645,510	-11,774,953	-660,557	19,088,658



Pairwise Dependency

Simulated IBNR for each pair of lines (one point per simulation), after copula coupling. A tilted cloud is positive dependence; a round cloud is near-independence.





Learn more about the benefits of this framework and how the BASR software is built to make implementing it in your reserving process as easy as possible at bedfordactuarial.com/basr



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